How can mathematics research increase effective instruction and student success?

Elementary and middle school students in the US are underachieving in mathematics. Those without adequate understanding of basic mathematical concepts and skills after completing kindergarten go on to struggle throughout their maths education. So how can teaching maths be improved to enable children to learn best? Dr Jonathan Brendefur and colleagues at the Developing Mathematical Thinking Institute (DMTI) have developed the five dimensions of the Developing Mathematical Thinking framework, a professional development programme for teachers and other educators. They demonstrate how teachers implementing the DMT approach benefit through increased content knowledge and student success.

The US National Center for Education Statistics (NCES) has reported that elementary and middle school students in the US are underachieving in mathematics. Additionally, their overall performance is not improving over time. In comparison with national standards, recent studies have revealed that these students are neither mathematically literate nor proficient. Studies have shown that those students completing kindergarten with an inadequate understanding of basic mathematical concepts and skills go on to struggle throughout their maths education. This research indicates that teaching methodologies need to change, particularly for students at the elementary level.

A central question then arises: how can teaching maths be improved to enable children to learn? Rising to this challenge, Dr Jonathan Brendefur and his colleagues at the Developing Mathematical Thinking Institute (DMTI) have spent years in classrooms working with teachers and students to perfect the five dimensions of the Developing Mathematical Thinking (DMT) framework, a multi-year professional development programme for improving maths instruction. The framework ensures that every teacher, parent, student, and administrator engages in successful maths experiences.

Dr Brendefur explains how teachers must establish 'equitable learning conditions that foster understanding' to enable students to become better problem-solvers both inside and outside the classroom. DMTI offers maths professional development, helping teachers apply the DMT framework to develop students' mathematical thinking. This involves building on students' ideas, encouraging problem-solving, and strengthening mathematical vocabulary, ultimately solidifying concepts by communicating mathematically.


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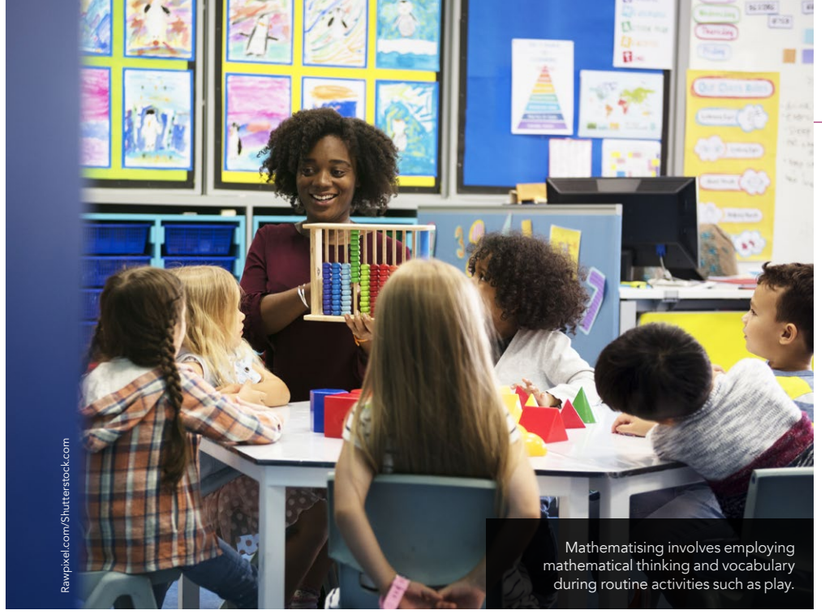
IMPROVING INSTRUCTIONAL PRACTICE AND STUDENT ACHIEVEMENT

Over the last decade, the DMTI team has been developing, and rigorously evaluating, the DMT framework. The research team has carried out longitudinal evaluative studies that demonstrate how the DMT programme affords students a better understanding of mathematics, reinforced with 'mathematising' and 'progressive formalisation' philosophies. Mathematising involves employing mathematical thinking and vocabulary during routine activities such as play. Through the process of progressive formalisation, an initial intuitive understanding of a basic concept is acquired. Based on this, more exact, formal connotations and increasingly complex theories can then be cultivated. For example, moving from manipulative cubes, to bar models, to symbolic notations.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Mathematising involves employing mathematical thinking and vocabulary during routine activities such as play.

The DMT framework's rubric comprises a five-dimensional approach to teaching mathematics. The researchers note, however, that these instructional practices are not mutually exclusive. The five dimensions are outlined below.

TAKING STUDENTS' IDEAS SERIOUSLY

Teachers initiate this first dimension by placing students in situations that engage their prior knowledge, enabling them to solve problems and expand their intuitive ideas. This is the onset of the development of mathematical understanding where educators should focus on processes of mathematising and progressive formalisation. Learning resources are required to provide teachers with options to differentiate groups of students and foster both their formal and informal knowledge.

ENCOURAGING MULTIPLE STRATEGIES AND MODELS

The second aspect encourages students to try out multiple models and strategies. Modelling is crucial in the development of mathematical thinking. As such, students need to be able to examine their chosen problem-solving method and compare it with alternative approaches to progress their understanding. The DMT framework includes carefully selected tasks that teachers can use with their students as they advance from their initial informal ideas to more conventional mathematical models.

Teachers must establish 'equitable learning conditions that foster understanding' to enable students to become better problem-solvers.

'I saw great enthusiasm, great ownership of what they were learning. I saw huge growth in their ability to reason and problem-solve.' Ann, 3rd-grade teacher.

'DMTI felt like that missing piece of how do you get kids to think mathematically? It really had tangible ways of the how.' Emily, 2nd-grade teacher.

PRESSING STUDENTS CONCEPTUALLY

This third component focuses on building connections between mathematical strategies and models, progressively formalising students' problem-solving concepts. Teachers aim to create environments that enable students to move beyond surface-level understanding of procedures to conceptualise mathematics and establish connections between various methods and models.

FOCUSING ON THE STRUCTURE OF MATHEMATICS

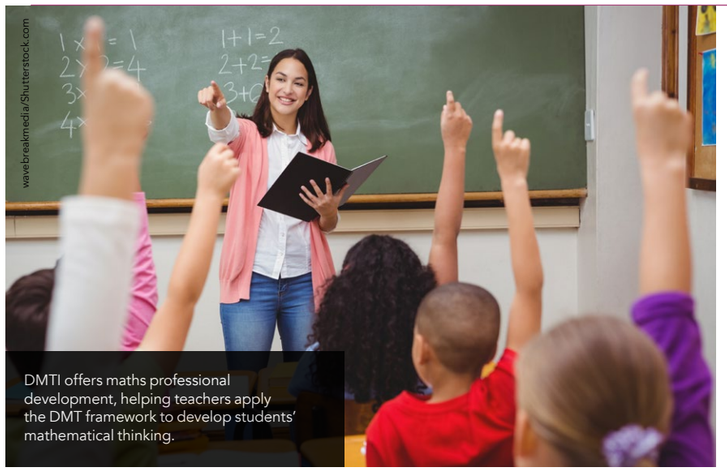
In this fourth element, 'structure' refers to the rudiments of mathematics that are constant throughout the subject regardless of the level being studied, for example, concepts of units, relationships, and equivalence. Shifting the focus to structure helps students understand and establish connections between concepts and entrenches structural component language relating to tasks throughout each lesson. The framework's formative assessment includes examples of students articulating and critiquing both their own and other students' models.

ADDRESSING MISCONCEPTIONS

In this fifth dimension, teachers use students' misconceptions and mistakes as tools to construct deeper levels of mathematical understanding. When a teacher focuses their practices on the four dimensions above, their attention shifts toward the informal strategies their students use for problem-solving, the mathematical connections they're making, the emerging conceptual understanding, and the structure of mathematics that brings about their misconceptions. By addressing these rather than ignoring them, teachers encourage their students to amend their thinking and make sense of maths. The framework embraces misconceptions and errors that can occur and encourages teachers to involve their students in justifying, evaluating, and inquiring into how they solve problems.

TEACHING FOR UNDERSTANDING

Dr Brendefur and his team have carried out longitudinal evaluative studies that demonstrate the effectiveness of the DMT pedagogic framework's five dimensions. They observed 268 lessons to find out how teachers' knowledge relates to their teaching of mathematics in terms of their



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DMTI offers maths professional development, helping teachers apply the DMT framework to develop students' mathematical thinking.

instructional practices. Analysis of the data collected over this six-year study revealed that providing teachers with professional development focused on increasing their students' mathematical thinking also has a beneficial impact on the teachers' knowledge. It was observed that those teachers who practised DMT in their classrooms increased their content knowledge as their instructional practices shifted towards teaching for understanding.

A review of the literature revealed a lack of rigorous quantitative research examining how teacher professional development correlates to students' achievement. To measure the effectiveness of the DMT programme, the team employed a cluster-randomised design. DMTI provides multi-tiered levels of professional development for teachers, so the team analysed how the minimal, moderate, and prominent levels affected teachers' instructional performance. They found that the achievement of those students whose teachers adopted all five dimensions increased the most. Furthermore, the programme positively impacted student achievement throughout the study.

STRATEGY VERSUS DRILL

Students' lack of multiplication fluency has been highlighted globally as a key factor that hinders mathematical performance. The researchers compared their DMT approach with traditional rehearsal and drilling practices in an experiment comparing the teaching of mathematics to grade 3, 4, and 5 students. Twelve teachers and 282 students from four schools with a history of below-average achievement took part in the study. Two schools formed the treatment group using the DMT strategy method, while the other two made up the comparison group employing the traditional drill technique.

The strategy group received fluency development instruction based on the DMT framework. The drill group received basic multiplication fluency instruction using methods favouring memorisation and repetition. At the beginning of the five-week unit students completed a pre-test made up of 30 multiplication questions. At the end of the unit, they completed the same questions in a post-test. The results demonstrated that while overall fact fluency is important, the performance of students in the strategy group increased much more than that of those in the drill group.

The more you know about the structure of something physically or theoretically the more you're able to adapt in practice situations.

BUILDING A STRONG FOUNDATION

Underpinned by their research into how we understand mathematics and structure mathematical ideas, the DMTI book, *Math Facts: Kids Need Them. Here's How to Teach Them* details and demonstrates the key components required for children to develop robust foundations for learning mathematics. It includes demonstration lessons and resources for teachers and other educators to use with elementary and middle school students. Offering guidance that builds on the five dimensions, it opens with modelling number facts using blocks and dice to develop maths fluency and continues through to multiplying non-integer rational numbers.

KEY COMPONENTS OF DMT

Exploration of mathematical concepts is promoted with the development of informal strategies while encouraging students to play with ideas. Students learn how and when to use more sophisticated models, having built a solid understanding of various models at their disposal. Students' conceptualisation is strengthened together with their understanding of mathematical procedures when they contrast different models and recognise that particular methods are more efficient than others.

Concentrating on mathematical structure supports connections, allowing students to gain a deeper understanding of establishing connections between the fundamental concepts, thus increasing their knowledge. Dr Brendefur comments that 'the more you know about the structure of something physically or theoretically the more you're able to adapt in practice situations.' Visual modelling enables students to understand why one strategy might work better than another. Misconceptions can be addressed and discussed during the modelling process, deepening students' levels of understanding so they can avoid such mistakes in the future.

The DTMI team has demonstrated that teachers implementing the DMT approach benefit from increasing their content knowledge as they shift their focus to teaching for understanding and increase their students' learning progressions and maths achievement.

Behind the Research

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Research Objectives

Dr Jonathan Brendefur at the Developing Mathematical Thinking Institute has devised the Developing Mathematical Thinking framework to improve maths education.

Detail

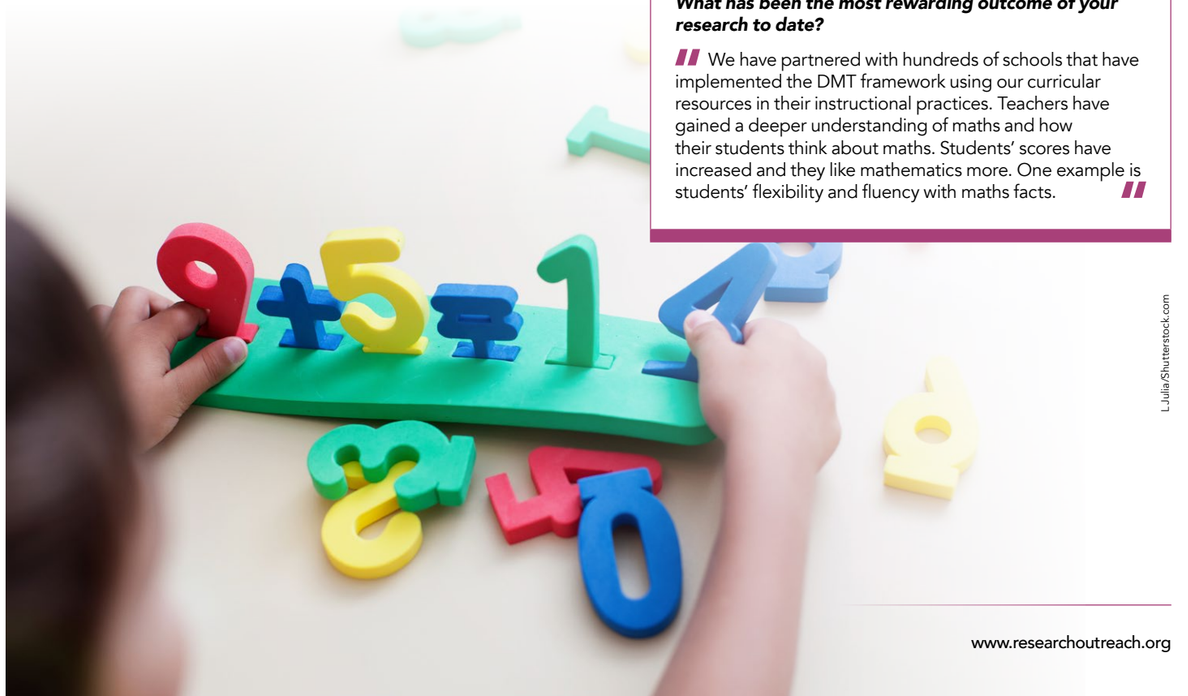
Bio

Jonathan Brendefur, PhD is the president of Developing Mathematical Thinking Institute, which partners with school districts to provide maths professional development and curricular resources to improve maths understanding and achievement. He is a former maths teacher and professor focusing on mathematical learning progressions, student learning, instruction, professional development, and curriculum.

Collaborators

- Sam Strother
- Jackie Ismail, PhD
- Keith Krone

DEVELOPING MATHEMATICAL THINKING INSTITUTE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Personal Response

What has been the most rewarding outcome of your research to date?

"We have partnered with hundreds of schools that have implemented the DMT framework using our curricular resources in their instructional practices. Teachers have gained a deeper understanding of maths and how their students think about maths. Students' scores have increased and they like mathematics more. One example is students' flexibility and fluency with maths facts."