How can mathematics research increase effective instruction and student success?

Elementary and middle school students in the US are underachieving in mathematics. Those without adequate understanding of basic mathematical concepts and skills after completing kindergarten go on to struggle throughout their maths education. So how can teaching maths be improved to enable children to learn best? Dr Jonathan Brendefur and colleagues at the Developing Mathematical Thinking Institute (DMTI) have developed the five dimensions of the Developing Mathematical Thinking framework, a professional development programme for teachers and other educators. They demonstrate how teachers implementing the DMT approach benefit through increased content knowledge and student success.

The US National Center for Education Statistics (NCES) has reported that elementary and middle school students in the US are underachieving in mathematics. Additionally, their overall performance is not improving over time. In comparison with national standards, recent studies have revealed that these students are neither mathematically literate nor proficient. Studies have shown that those students completing kindergarten with an inadequate understanding of basic mathematical concepts and skills go on to struggle throughout their maths education. This research indicates that teaching methodologies need to change, particularly for students at the elementary level.

A central question then arises: how can teaching maths be improved to enable children to learn best? Dr Jonathan Brendefur and colleagues at the Developing Mathematical Thinking Institute (DMTI) have developed the five dimensions of the Developing Mathematical Thinking framework, a multi-year professional development programme for improving maths instruction. The framework ensures that every teacher, parent, student, and administrator engages in successful maths experiences.

Dr Brendefur explains how teachers must establish ‘equitable learning conditions that foster understanding’ to enable students to become better problem-solvers. This involves building on students’ ideas, encouraging problem-solving, and strengthening mathematical vocabulary, ultimately solidifying concepts by communicating mathematically.

IMPROVING INSTRUCTIONAL PRACTICE AND STUDENT ACHIEVEMENT

Over the last decade, the DMTI team has been developing, and rigorously evaluating, the DMT framework. The research team has carried out longitudinal evaluative studies that demonstrate how the DMT programme affords students a better understanding of mathematics, reinforced with ‘mathematising’ and ‘progressive formalisation’ philosophies. Mathematising involves employing mathematical thinking and vocabulary during routine activities such as play. Through the process of progressive formalisation, an initial intuitive understanding of a basic concept is acquired. Based on this, more exact, formal connotations and increasingly complex theories can then be cultivated. For example, moving from manipulative cubes, to bar models, to symbolic notations.

The DMT framework’s rubric comprises a five-dimensional approach to teaching mathematics. The researchers note, however, that these instructional practices are not mutually exclusive. The five dimensions are outlined below:

1. TAKING STUDENTS’ IDEAS SERIOUSLY
   Teachers initiate this first dimension by placing students in situations that engage their prior knowledge, enabling them to solve problems, and expand their intuitive ideas. This is the onset of the development of mathematical understanding where educators should focus on processes of mathematising and progressive formalisation. Learning resources are required to provide teachers with options to differentiate groups of students and foster both their formal and informal knowledge.

2. ENCOURAGING MULTIPLE STRATEGIES AND MODELS
   The second aspect encourages students to try out multiple models and strategies. Modelling is crucial in the development of mathematical thinking. As such, students need to be able to examine their chosen problem-solving method and compare it with alternative approaches to progress their understanding. The DMT framework includes carefully selected tasks that teachers can use with their students as they advance from their initial informal ideas to more conventional mathematical models.

3. PRESSING STUDENTS CONCEPTUALLY
   This third component focuses on building connections between mathematical strategies and models, progressively formalising students’ problem-solving concepts. Teachers aim to create environments that enable students to move beyond surface-level understanding of procedures to conceptualise mathematics and establish connections between various methods and models.

4. ADDRESSING MISCONCEPTIONS
   In this fifth dimension, teachers use students’ misconceptions and mistakes as tools to construct deeper levels of mathematical understanding. When a teacher focuses on their practices on the four dimensions above, their attention shifts toward the informal strategies their students use for problem-solving, the mathematical connections they’re making, the emerging conceptual understanding, and the structure of mathematics that brings about their misconceptions. By addressing these rather than ignoring them, teachers encourage their students to amend their thinking and make sense of maths. The framework embraces misconceptions and errors that can occur and encourages teachers to involve their students in justifying, evaluating, and inquiring into how they solve problems.

5. TEACHING FOR UNDERSTANDING
   Dr Brendefur and his team have carried out longitudinal evaluative studies that demonstrate the effectiveness of the DMT pedagogic frameworks five dimensions. They observed 268 lessons to find out how teachers’ knowledge relates to their teaching of mathematics in terms of their
Behind the Research

Dr Jonathan Brendefur, PhD

DMT offers maths professional development, helping teachers apply the DMT framework to develop students’ mathematical thinking.

The more you know about the structure of something physically or theoretically the more you’re able to adapt in practice situations.

Research Objectives

Dr Jonathan Brendefur at the Developing Mathematical Thinking Institute has devised the Developing Mathematical Thinking framework to improve maths education.

Detail

Bio
Jonathan Brendefur, PhD is the president of Developing Mathematical Thinking Institute, which partners with school districts to provide maths professional development and curricular resources to improve maths understanding and achievement. He is a former maths teacher and professor focusing on mathematical learning progressions, student learning, instruction, professional development, and curriculum.

Collaborators
- Sam Strother
- Jackie Isem, PhD
- Keith Krone

References


Brendefur, J, Strother, S, (2021) Here’s how to teach them. DMTI Incorporated, Boise, Idaho


Personal Response

What has been the most rewarding outcome of your research to date?

We have partnered with hundreds of schools that have implemented the DMT framework using our curricular resources in their instructional practices. Teachers have gained a deeper understanding of maths and how their students think about maths. Students’ scores have increased and they like mathematics more. One example is students’ flexibility and fluency with maths facts.

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