Exploring transitions to chaos in complex systems

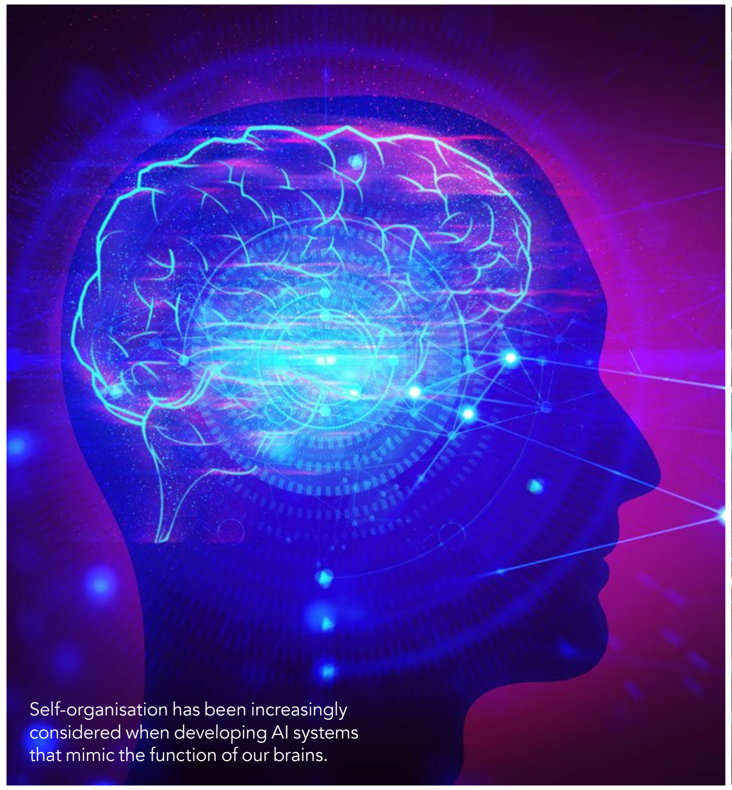
Previously, Dr Alberto Robledo of Instituto de Física, Universidad Nacional Autónoma de México (UNAM) showed how transitions to chaos in nonlinear systems can be expressed in the language of statistical mechanics. Additionally, in his studies he shows how the same mathematical laws can link these transitions to the behaviours of different types of complex systems. His results may lead to new discoveries about many areas of nature where complex systems can be found.

Complex systems are found across a diverse array of situations in nature: from human societies to groups of interacting quantum particles. They describe collections of interconnected and interdependent elements, which interact with each other in highly unpredictable ways and are therefore incredibly difficult to describe.

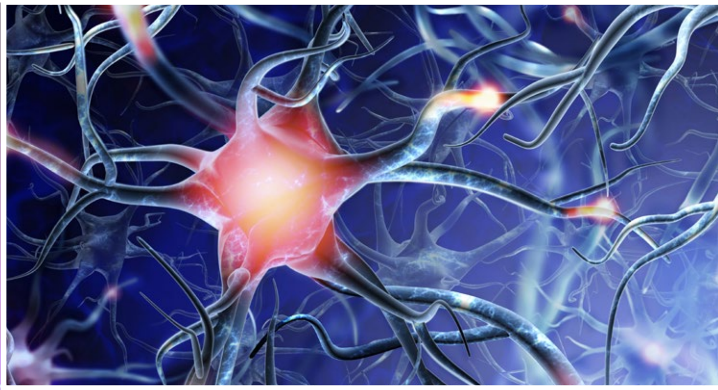
Unlike systems such as electrical circuits, simple chemical reactions, and gases such as air, researchers can't simply predict how complex systems will behave using basic mathematical laws. Instead, they must be studied with an empirical approach, meaning researchers collect data on the system and perform statistical analysis to identify patterns, correlations, and trends, which may hint at the complex systems' underlying mechanisms that lead to properties that do not belong to the individual components themselves. Based on this information, researchers will often build models to reproduce the behaviour they observe as accurately as possible.

As Dr Alberto Robledo of Universidad Nacional Autónoma de México explains, the need to understand how complex systems behave has led to an explosion of recent research – but so far, these studies have been somewhat limited in their scope. 'The study of complex systems constitutes a new and interdisciplinary research area that cuts across physics, biology, ecology, economics, sociology, and the humanities', he says. 'To date, most studies in this field have had an empirical character, despite the many attempts to advance understanding at the next, phenomenological level.'

This means that in focusing on the small scales outside those in which the underlying mechanisms of complex systems play out, researchers haven't been able to accurately describe their observable, large-scale behaviours.



Self-organisation has been increasingly considered when developing AI systems that mimic the function of our brains.



TRANSITIONS INTO CHAOS

Through his research, Robledo aims to advance this understanding by focusing on one of the most critical aspects of complex systems – 'nonlinear' effects. As we saw previously, this behaviour relates to systems whose responses aren't directly proportional to inputs from their surrounding environments.

More specifically, he explores the transitions of nonlinear systems into a state of chaos – borderline states governed by deeply intricate webs of underlying patterns, interconnections, and repeating feedback loops. In contrast, when behaviour is already chaotic, randomness is always present. Those at the transitions to chaos display instead long-memory recurrent arrangements. To briefly recap, these transitions can occur along three known routes: named 'intermittency', 'period doubling', and 'quasi-periodicity'.

Robledo studies this marginal behaviour between order and chaos using the language of statistical mechanics. We recollect that the latter is a branch of physics that aims to understand and predict how thermodynamic systems with large numbers of particles will evolve over time. It treats the collective motion of each atom and molecule in a system as a collection of possible microstates or configurations, each with a certain probability of occurring.

Robledo aims to establish a basic link between the many existing models of nonlinear dynamics with complex systems.

By analysing these probabilities, scientists can make predictions about different properties of the system, including its temperature, pressure, and entropy – a measure of its randomness and disorder, as it is the case for chaotic regimes in nonlinear dynamics. Researchers use statistical mechanics to model thermodynamic systems – but when chaos is at the brink of being removed, things become far more complex.

To study the behaviours of complex systems, Robledo again enlists the help of the 'low-dimensional nonlinear iterated maps' we encountered previously. These mathematical constructions describe the evolution of systems via a relatively small number of variables over discrete steps in time, making them ideal for creating and studying simple models capable of capturing complex phenomena.

DESIGNING NEW MODELS

With this platform, Robledo examines the abundant 'scaling laws' of transitions into chaos, which describe the characteristic patterns and behaviours of many different types of complex systems in mathematical terms. 'Our general purpose here has been to advance simple existing models, or to design new models, that allow us to address basic issues about complex systems, and to predict their behaviours merely by observing them perform', Robledo describes.

Ultimately, Robledo aims to establish a basic link between the many existing models of nonlinear dynamics with complex systems and to express

Self-organisation is often seen in nature: in the synchronised movements of flocks of birds and schools of fish and in the orderly patterns of snowflakes and honeycombs.

them in the basic language of statistical mechanics.

A ZODIAC OF STUDIES

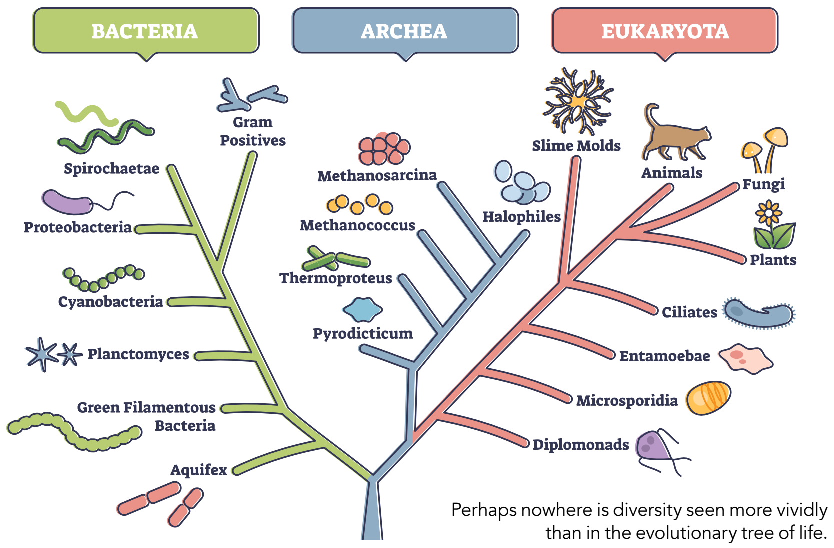
Building on his previous work, Robledo carried out what he calls a 'zodiac' of 12 studies on complex systems. Each of these studies focused on a different observable phenomenon resulting from transitions into chaos in different types of complex systems.

Together, Robledo hopes these studies will guide researchers away from a purely empirical approach to studying complex systems via nonlinear dynamical models and encourage them to investigate the underlying laws of their observable, shared, large-scale behaviours.

Some of the examples of nonlinear dynamics considered in Robledo's zodiac are becoming increasingly relevant in studies of the natural world and its impact on our own lives. Some of them are outlined below.

SELF-ORGANISATION

Self-organisation is often seen in nature in the synchronised movements of flocks of birds and schools of fish, in the orderly patterns of snowflakes and honeycombs, and in the ability of the neurons in our own brains to learn and process information.



Perhaps nowhere is diversity seen more vividly than in the evolutionary tree of life.

In recent years, it has been increasingly considered when developing artificial intelligence systems, which mimic the function of our brains. Elsewhere, researchers are using models of self-organisation to design devices whose properties are far more advanced and complex than those of their simple component parts.

Robledo's work shows that the approach to the transitions into chaos can cause self-organised structures to emerge spontaneously without any external control. As a result, modelling this behaviour can be crucial to understanding how complex patterns materialise in nature, as well as their relevance in modern technology.

DIVERSITY

Perhaps nowhere is diversity seen more vividly than in the evolutionary tree of life. Here, complexity emerges as new species branch off from a single common ancestor and rapidly diversify to exploit new ecological niches. Not only can this cause organisms to develop highly sophisticated structures and behaviours to survive and reproduce; it can also cause entirely different species to form symbiotic relationships, where each depends on the other for their survival.

Robledo has pointed out a link between Tsallis entropy evolution at the transitions to chaos and the measure of growth of diversity.

EVOLUTIONARY GAME THEORY

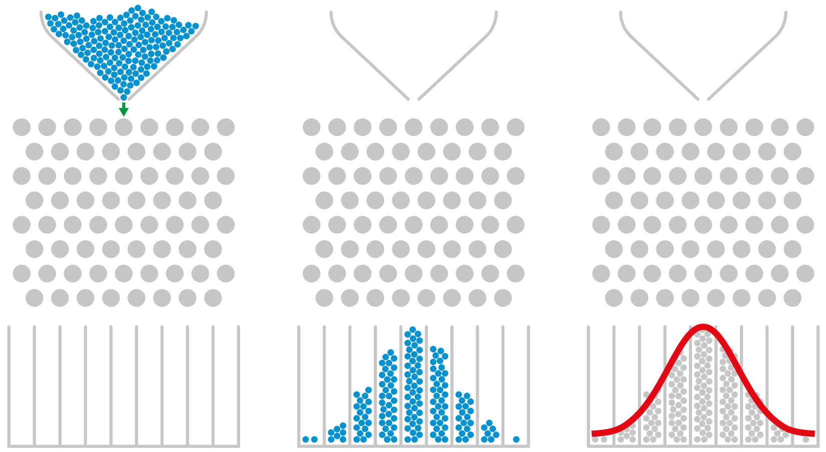
Game theory is widely used by mathematicians to study the strategies that emerge in complex systems where rational minds interact and compete. Since the interplay of different game strategies is so complex, usually only simple outcomes are available.

Robledo modified the ordinary game theory approach to exhibit a link with nonlinear iterated maps and the incorporation of more elaborate regular and chaotic behaviours as well as the transitions between them. In consequence, the enriched method is particularly relevant when studying the strategic interactions taking place between co-evolving species in situations where the survival of one species depends on the evolutionary strategies taken by others it interacts with.

THE ONSET OF CHAOS AND CRITICALITY

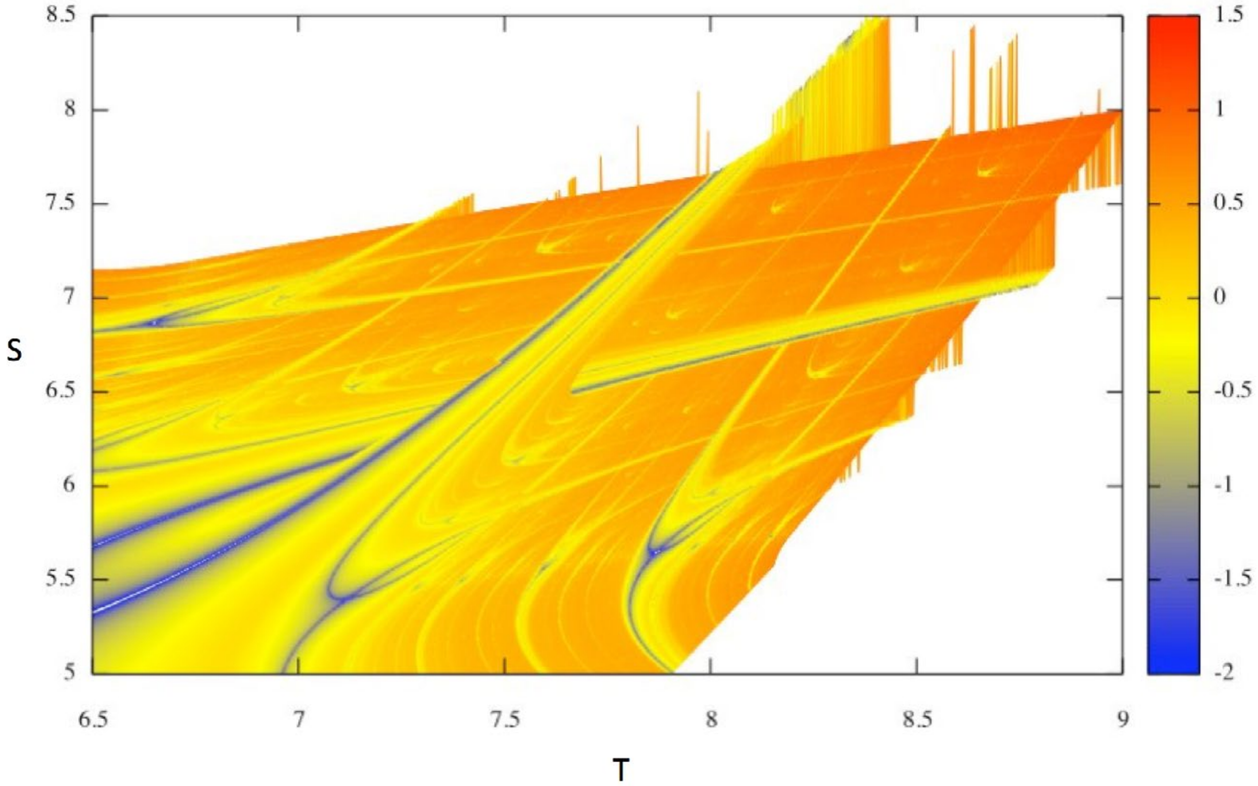
Beyond these mainly biological and ecological examples, Robledo's zodiac also includes more general studies of how marginally chaotic systems could be considered theoretically on larger domains.

The onset of chaos describes the exact point at which a nonlinear system transitions from exhibiting regular, predictable behaviour to chaotic and



The bell-shaped Gaussian curve describes the probability distribution of many real-world variables.

Chaos in Game Theory



Lyapunov exponent as a function of social dilemmas *T* (cooperate) and *S* (defect). Orange denotes chaos; yellow the transitions to chaos.

unpredictable behaviour. In condensed matter physics, a point of criticality is a special state that signals the endpoint of, say, liquid and gas distinction, a state at which there are variations at all scales. Robledo's work has shown the equivalence between these two borderline concepts that have become paradigms for the study of the scaling laws observed in complex system phenomena. Considering these boundary concepts is especially important when modelling situations, including not only liquids and gases but natural ecosystems and even financial markets.

CENTRAL LIMIT STATIONARY DISTRIBUTION

In the context of the theory of probability, the 'central limit theorem' predicts that under certain general conditions, the probabilities of many sets of data from many sources will tend to follow a normal distribution as the size of the samples increase – the bell-shaped curve describing the probability distribution of many real-world variables. As their number grows, the evolving shape of the distribution of the sum of sample data will converge to that of the normal distribution.

Based on previous knowledge that the central limit theorem applies to the sums of states or points in chaotic dynamics, Robledo has clarified the crossover transformation of the normal distribution into a fractal stationary distribution at the onset of chaos.

RANKED DATA AND ALLOMETRY

It is common when examining data from complex systems, like sizes of cities or frequencies of words in texts, to sort or 'rank' the data. In doing this, important empirical laws have surfaced as they exhibit universal scaling behaviours. This is also the case when ordering biological data, such as metabolic rates, in terms of the organisms' or species' increasing sizes or masses. The scaling relationships between physical properties and the individuals' overall size are examples of 'allometry'. Over the years, researchers have been puzzled by these remarkably predominant patterns and advanced plausible explanations.

Robledo has moved forward towards a deeper insight into these laws by showing a close link of them with a transition to chaos, revealing that the successive stages of the dynamics at this borderline condition reproduces quantitatively all kinds of these data. That is, the scaling laws omnipresent in real ranked data coincide with those at the onset of chaos.

CRITICAL FLUCTUATIONS

One final area of research where Robledo's zodiac of studies is particularly relevant is in condensed matter physics. Here, he explores several major problems regarding complexity in hard-to-solve statistical-mechanical problems; one instance is critical fluctuations.

As a difference with all other equilibrium states, at critical points, physical quantities fluctuate at all scales around their average, measurable values. This unique fact constitutes an exceptional window into the inner scales of macroscopic systems. Fluctuations are evanescent features that appear one after the other as earlier fluctuations collapse. Robledo has described them via the intermittency displayed in the neighbourhood of the transition to chaos via the same name route out of chaos. A possible application to complex systems in other fields is the elaboration of a method to obtain early warnings or tipping points for catastrophic events.

LOCALISATION TRANSITIONS

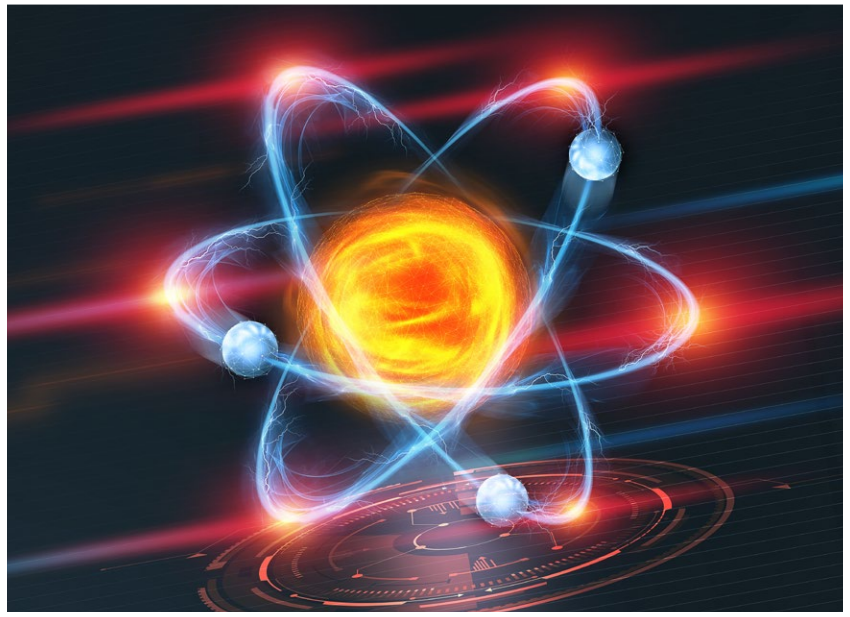
Perhaps the best-known phenomenon of a localisation transition is the transformation of a conductor into an insulator or vice versa. Electrons as waves undergo scattering when they traverse materials, and as a result they can adopt two different behaviours: become localised (individually restricted) or act coherently (flow cooperatively). Similarly, light and sound can also exhibit localisation transitions as other instances of wave media susceptible to scattering.

Looking at a mathematical model capable of reproducing the localisation transition, Robledo found a robust analogy with the dynamical properties of a nonlinear iterated map exhibiting transitions to chaos via the intermittency route. This analogy unveiled a novel connection between the localisation phenomenon in physics and a mathematical formulation of the onset of chaos. Furthermore, it implied too another instance for the incidence of Tsallis entropy.

GLASSY DYNAMICS

Rapid cooling of liquids past the crystallisation temperature leads to

Robledo hopes his studies will encourage researchers to investigate the underlying laws of the complex systems' observable, shared, large-scale behaviours.



Statistical mechanics treats the collective motion of each atom and molecule in a system as a collection of possible microstates.

glass formation. At low temperatures, molecules lack the necessary energy for collective rearrangements and an amorphous solid materialises. The special scaling geometry of trajectory positions generated by a nonlinear iterated map at the period-doubling transition to chaos could be gradually disordered by the addition of a random variable term, called 'noise'. Robledo realised that by considering the noise amplitude in this mathematical setting to play the role of temperature in the cooled liquid, the main features of glass formation could be soundly imitated. A hidden relationship between glasses and nonlinear systems at a transition to chaos was revealed, which could also be applied in various situations like traffic jams.

LINKS TO STATISTICAL MECHANICS

Through his zodiac of studies, Robledo shows how the leading properties of complex systems can be linked together in all these diverse scenarios, by means of the same scaling laws displayed at the transitions to chaos in nonlinear iterated maps.

Even further, his results tie in with his previous examination of Tsallis entropy: the expression for an enhanced statistical mechanics we encountered previously. 'Interestingly, the scaling laws that govern transitions to chaos are reflected in the properties of our models as characteristic elements of Tsallis statistics', Robledo says.

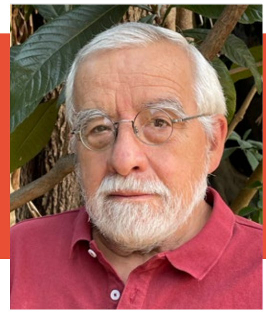
As we saw previously, Robledo's research has proven a basic link between nonlinear dynamics at the point where systems transition into chaos, and the fundamental principles of statistical mechanics. 'We have now been able to advance simple models that correctly predict features of some of the main issues in the field of complex systems', Robledo continues. 'These models have one nonlinear dynamical characteristic in common: the transition to chaos in iterated maps.'

In other words, the dynamics detailed in these models can accurately describe the empirically observable processes taking place in many different types of complex systems and show how they can be expressed in the language of statistical mechanics through Robledo's approach to obtain the Tsallis entropy formula.

The dynamics detailed in these models can accurately describe the processes taking place in many different types of complex systems.

Robledo now hopes that his zodiac of studies will inspire researchers to take a new approach to understand how complex systems generate their scale-free properties we observe. Ultimately, this may lead to new discoveries about the many phenomena in nature, which are driven by nonlinear dynamics at their most interesting situation, the onset of chaos.

Complex systems can be found across a diverse array of situations in nature.



Behind the Research

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Research Objectives

Dr Alberto Robledo studies transitions to chaos in different types of complex systems.

Detail

Bio

Dr Alberto Robledo is a senior research scientist at Instituto de Física, Universidad Nacional Autónoma de México (UNAM). Robledo earned his undergraduate degree from UNAM and his doctorate from the University of St. Andrews, UK. He has conducted extensive research in the fields of statistical physics and complex systems for over fifty years.

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Personal Response

How did you first recognise the need to better understand complex systems at a phenomenological level?

" I think this has been evident already for some time to many colleagues in the field. There exist large sets of available real data associated with many different complex systems in various disciplines. In contrast, there is absence of comprehensive laws other than a few of empirical origin. The attempts for advancing the understanding of general patterns of behaviour have been based on a range of methodologies that imply different mechanisms, while universal principles have remained basically as guiding paradigms, such as the 'edge of chaos' or 'criticality'.

What do you think is the most interesting example of a transition into chaos in a complex system?

My own point of interest is how diverse problems, disconnected at first sight, adopt explanations about different concerns based on a common theoretical formalism or approach. Perhaps a challenging example is that of self-organisation, observed in nature but perceived to run against the laws of thermodynamics. The aim was to design a simple model that could be 'run' like a computer program and observe an unpretentious instance of self-organisation. The period-doubling cascade into chaos provided one answer.

Do you think your results could help Tsallis entropy to become a more widely accepted theory?

All proposed scientific advances permeate our community when actual evidence and understanding is provided. This is particularly true for the physical sciences for which eventually available formal proofs and verifiable quantitative predictions are requested. The most interesting episodes are perhaps those where definite answers are not obvious and a saga develops before solutions appear from unexpected places. Places that often were first developed, in say, mathematical themes, with other motivations in mind, or in other cases only recently established. "